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Tachyonic properties of space- and time-trapped electromagnetic fields

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Abstract. Space and time cavities with internally trapped electromagnetic fields are considered as wave–particle models of extended time-like and space-like particles. It is shown that in an N-dimensional ($N = 1, 2, 3$) space-cavity photons undergo a conversion into a system composed of a bradyon at rest and N infinite-speed *transcendent* tachyons moving circularly about the bradyonic constituent. The time-like Klein–Gordon equation and its space-like counterpart for inner and outer fields associated with trapped radiation have been derived.

1. Introduction

The internal structure of massive particles remains one of the most important and fascinating problems to be solved by contemporary science. Standard conceptualization in physics usually attributes a point-like character to material objects; however, this approach leads to the unphysical effects of infinite self-energy and self-field, so that Dirac distributions or a renormalization procedure must be used to overcome the above difficulties. In addition, at the semiclassical level attempts were made to obtain a finite mass of point particles by employing the Casimir effect (Casimir 1956, Boyer 1968, Milton 1980); however, without satisfactory results.

Recent developments in this field have brought forth several interesting concepts in which particles are treated as extended structures and not as point-like objects. For example, Jehle (1971, 1972, 1975) considered leptons, quarks and hadrons in terms of electromagnetic fields and their amplitude distributions, proving that the topological structure of these fields represents the internal quantum numbers in particle physics. Post (1982) showed that the properties of an elementary charge indicate an identification with a closed hypersurface in spacetime. The electron in this picture emerges as a ring current, involving the elementary charge circulating azimuthally as a tubular sheath of distributed charge (Post 1986).

Electromagnetic models in which a massive particle is considered as a relativistic phase-locked cavity with an internally trapped radiation (Jennison and Drinkwater 1977, Jennison 1978, 1980, 1983, 1986), or as associated with a system of *luminal* velocity standing waves obtained by superposition of two running waves propagating in opposite directions (Elbaz 1983, 1985, 1986a, b, Kostro 1985) have also been proposed. In particular, it has been shown (Jennison and Drinkwater 1977) that a phase-locked cavity with an internal standing electromagnetic wave has the inertial properties of a particle, and *vice versa* all particles endowed with inertial mass may be considered as trapped radiation.

A combination of the last two approaches leads to the photon concept of internal particle structure (Molski 1991), in which *longitudinal* photons associated with electromagnetic fields trapped in phase-locked cavities and waveguides are considered as a relativistic model of a massive particle. Such photons, contrary to *transverse* photons, are endowed with a non-zero rest mass (which depends on the form of waveguide and the type of wave propagation), and move in the *subluminal* velocity range. Application of the electromagnetic approach permits the construction of extended particle models and a better understanding of their internal structure. For example, it has been demonstrated (Molski 1991) that the radiation trapped in a phase-locked cavity undergoes a conversion into a system of time- and space-like waves which *lock* to form the photon-like wave. In the corpuscular picture it may be interpreted as photon conversion into a system composed of both bradyonic and tachyonic components which trap each other in a relativistically invariant way, giving a compound particle of the photon-like type.

In order to obtain a clear and complete picture of the electromagnetic cavity interior, the fundamental question arises: what is the internal structure of such a bradyon–tachyon compound or, in other words, how are they related to each other topologically?

Another important problem appears in the methodological framework of deriving the space-like Proca equation (Molski 1991), which describes propagation of a tachyonic vector field associated with the space-like component of longitudinal photons. A detailed analysis of the problem leads to the conclusion that the photon model predicts the presence of space-like waves in the spectrum of matter waves associated with a cavity interior; however, it does not argue for or against the existence of space-like waves in an external cavity domain. The matter waves of second kind (D-waves) outside the cavity can only be postulated to exist, and then introduced in a formal manner by using the correspondences between the ordinary de Broglie B-wave and the D-wave (Molski 1991). Such a derivation of the space-like Proca equation, in contradiction to the time-like one, seems to be artificial and unsatisfactory.

The intention of this paper is to consider the above problems in the framework of the photon model of a massive particle (Molski 1991), and the tachyonic theory of elementary particle structure (Corben 1977, 1978a, b, Castorina and Recami 1978, Rosen and Szamosi 1980, Recami 1986) involving the *two-wave* particle model (Horodecki 1982, 1983, 1984, 1988a, b, Das 1984, 1986, 1988, Elbaz 1983, 1985, 1986a, b, 1987, 1988) as its special case. In particular, we shall be concerned with a generalization of the photon model to include the case of radiation trapped in cuboid space and time cavities. Our investigations indicate the possibilty of deriving the time-like Klein–Gordon equation and its space-like counterpart for inner and outer fields associated with a free-moving electromagnetic cavity treated as an extended massive object.

2. Space-trapped electromagnetic fields

Let us consider an N-dimensional ($N = 1, 2, 3$) electromagnetic space cavity (S^N-cavity) of dimensions a_α ($\alpha = 1, \dots, N$), with perfectly conducting walls and being charge-free inside. The famous Maxwell equations

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho \qquad \nabla \times \boldsymbol{H} - 4\pi c^{-1}\boldsymbol{j} - c^{-1}\frac{\partial \boldsymbol{E}}{\partial t} = 0 \tag{1}$$

$$\nabla \cdot \boldsymbol{H} = 0 \qquad \nabla \times \boldsymbol{E} + c^{-1}\frac{\partial \boldsymbol{H}}{\partial t} = 0 \tag{2}$$

are valid both for electromagnetic inner fields inside a charged particle, as well as for the outer fields responsible for particle interactions (Elbaz 1987, 1988). The electromagnetic fields propagate according to the equations

$$\partial_\mu \partial^\mu \boldsymbol{E} + 4\pi\left(c^{-2}\frac{\partial \boldsymbol{j}}{\partial t} + \nabla\rho\right) = 0 \tag{3}$$

$$\partial_\mu \partial^\mu \boldsymbol{H} + 4\pi c^{-1}\nabla \times \boldsymbol{j} = 0 \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{j} = 0 \tag{5}$$

which are a direct consequence of the Maxwell equations (1) and (2).

As the electric charge is fully *absorbed* by a material particle, the outer electromagnetic field is characterized by $\rho = \boldsymbol{j} = 0$. However, considering the inner fields of a charged particle, the charge and current densities do not vanish and must be taken into account. For an electromagnetic cavity with a charge-free interior, it may be assumed that $\rho = \boldsymbol{j} = 0$, so the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy the *luminal* Maxwell equation

$$\partial_\mu \partial^\mu \Psi(x^\mu) = 0 \qquad \mu = 0, 1, 2, 3 \tag{6}$$

valid for inner and outer fields. In this case the electric charge is distributed all over a thin pellicula which constitutes the wall of the particle, which in our approach corresponds to the wall of the cavity.

The last equation for the S^N-cavity can be given in the general form

$$\partial_\mu \partial^\mu \phi_{n_\alpha}(x^\alpha) \exp[i(k_\mu x^\mu)] = 0 \qquad \mu \neq \alpha = 1, \ldots, N \tag{7}$$

where $k^\mu \equiv (\omega/c, \boldsymbol{k})$, $x^\mu \equiv (ct, \boldsymbol{r})$ are the wave 4-vectors and the position 4-vectors, respectively, whereas the functions $\phi_{n_\alpha}(x^\alpha)$ are solutions of the Helmholtz equation

$$(\Delta + m^2_{n_\alpha})\phi_{n_\alpha}(x^\alpha) = 0 \qquad n_\alpha = 0, 1, 2 \ldots. \tag{8}$$

Taking into account the boundary conditions for the fields $\boldsymbol{E}$ and $\boldsymbol{H}$, the solutions of the Helmholtz equation have the form (Jackson 1975)

$$\phi_{n_\alpha}(x^\alpha) = H_0 \prod_{\alpha=1}^{N} \cos\left(\frac{\pi n_\alpha x^\alpha}{a_\alpha}\right) \tag{9}$$

$$\phi_{n_\alpha}(x^\alpha) = E_0 \prod_{\alpha=1}^{N} \sin\left(\frac{\pi n_\alpha x^\alpha}{a_\alpha}\right) \tag{10}$$

$$m^2_{n_\alpha} = \omega^2 c^{-2} = \pi^2 \sum_{\alpha=1}^{N} \frac{n^2_\alpha}{a^2_\alpha}. \tag{11}$$

In the light of the results obtained in our previous work (Molski 1991), a non-zero rest mass

$$m^0_{n_\alpha} = \hbar m_{n_\alpha} c^{-1} = \hbar \pi c^{-1} \left(\sum_{\alpha=1}^{N} \frac{n_\alpha^2}{a_\alpha^2} \right)^{1/2} \tag{12}$$

may be attributed to electromagnetic fields imprisoned in an S^N-cavity. Such an *associated mass has the inertial properties of ordinary ponderable matter with the* possibility of quantization of different inertial states (Jennison and Drinkwater 1977, Jennison 1978), so intrinsically stable massive objects may be considered as the modes of imprisoned radiation. In view of the above an electromagnetic S^3-cavity very well reproduces the fundamental properties of time-like objects, i.e. ponderability, inertiality, non-point character and three-dimensional extension, hence it may be employed as an electromagnetic model of extended subluminal particles.

3. Tachyonic properties of space-trapped inner fields

The idea that tachyons (space-like, faster-than-light objects) may play a role in elementary particle structure has been taken up by many authors (Recami 1968, 1969, Guenin 1976, Hamamoto 1972, 1974, Akiba 1976, Rafanelli 1974, 1976, 1978, Van der Spuy 1978, Maccarrone and Recami 1980, Mignani and Recami 1975). In particular it has been suggested (Corben 1977, 1978a, b, Castorina and Recami 1978, Rosen and Szamosi 1980, Recami 1986) that some particles may be composite objects built up of both bradyonic and tachyonic components.

Let us recall here the most important results relevant in the framework of our considerations. A free bradyon (time-like, slower-than-light object) with a rest mass m_0 and a free tachyon with a rest mass m'_0 can trap each other in a relativistically invariant way yielding a compound particle of rest mass $M_0 = (m_0^2 - m_0'^2)^{1/2}$, provided that $m_0 > m'_0$. Such a composite object is described by the wavefunction $\Psi = \psi\psi'$ satisfying the wave equation $(\partial_\mu \partial^\mu + (M_0 c/\hbar)^2)\Psi = 0$, with respect to the invariant interaction condition $\partial_\mu \psi \partial^\mu \psi' = 0$. For $m_0 = m'_0$, the compound particle has photon-like characteristics, and the tachyonic theory reduces to the *two-wave* particle model (Horodecki 1982, 1983, 1984, 1988a, b, Das 1984, 1986, 1988, Elbaz 1983, 1985, 1986a, b) included as a special case (Molski 1991).

Bearing in mind that imprisoned photons undergo conversion into a bradyon–tachyon system (Molski 1991), it would be tempting, on the basis of the above concept, *to penetrate* the cavity interior and to investigate its internal wave–particle structure. In order to accomplish this, let us assume that an S^N-cavity is at rest relative to the laboratory system (radiation inside the cavity is not Doppler shifted): then for inner fields the wave equation (6) may be rewritten into the equivalent formula

$$\left[\partial_\mu \partial^\mu + m^2_{n_\alpha} - \sum_{\alpha=1}^{N} (\pi n_\alpha a_\alpha^{-1})^2 \right] \Psi(x^\mu) = 0 \tag{13a}$$

$$\Psi(x^\mu) = \psi(x^0)\psi(x^\alpha)' \tag{13b}$$

where $m^0_{n_\alpha} = m_{n_\alpha} \hbar c^{-1}$ is the rest mass of trapped radiation, and $\psi(t)$, $\psi(x^\alpha)'$ are bradyonic and tachyonic inner fields associated with the cavity at rest. Application of the variable separation method decomposes (13*a*) into the set of two wave equations

$$(\partial_0^2 + m_{n_\alpha}^2)\psi(x^0) = 0 \tag{14a}$$

$$\left[\partial_\alpha \partial^\alpha + \sum_{\alpha=1}^{N} (\pi n_\alpha a_\alpha^{-1})^2\right] \psi(x^\alpha)' = 0. \tag{14b}$$

The solution of (14*a*) is periodic in time and independent of position so it describes a bradyonic component at rest, whereas, in contrast, the solution of (14*b*) is static in time and periodic in space, since it corresponds to an infinite-speed tachyonic constituent termed a *transcendent* tachyon (Recami 1986 p 23). Carrying out complete separation of spatial variables in (14*b*) one obtains the set of N one-dimensional equations

$$[\partial_\alpha^2 + (\pi n_\alpha a_\alpha^{-1})^2]\psi(x^\alpha)' = 0 \qquad \alpha = 1, \ldots, N \tag{15}$$

$$\psi(x^\alpha)' = \exp[\pm \mathrm{i}(\pi n_\alpha a_\alpha^{-1} x^\alpha)] \tag{16}$$

whose solutions correspond to the periodical motion along the circular trajectory of a radius $r_\alpha = a_\alpha \pi^{-1}$ and a circumference $d_\alpha = 2a_\alpha$.

The above results permit us to obtain the wave–particle picture of an S^3-cavity interior with internally trapped electromagnetic fields. Namely, the solutions (16) describe the N *transcendent* tachyons moving circularly about the x^α-axes whereas the circle centres coincide with a bradyonic component at rest. If we assume that the constituents of a compound particle interact through a field similar to the gravitational one (Recami 1982), then the infinite-speed motion of a *transcendent* tachyon corresponds to the ground state of the *Kepler-type* system of particles, with the trapped force holding the tachyon on a circular orbit tending to zero (Recami 1984). In view of the above, a compound particle may be viewed as a couple of free particles (Recami 1985, 1986 p 112).

In the wave picture, the $\Psi(x^\mu) = \psi(x^0)\psi(x^\alpha)'$ may be interpreted as a superposition of time- and space-like fields which *lock* to form the photon-like field. In the case of an S^3-cavity, the interaction between such fields is analogous to the scattering of a wave by a diffracting grating (Corben 1978a). The three values of the *lattice spacings* in the three directions of space correspond to the masses of the three space-like states that can combine with one time-like state (Corben 1978b, Recami 1986 p 113). The above tachyonic interpretation of an internal S^N-cavity structure is in full agreement with Corben (1977, 1978a, b), Castorina and Recami (1978), as well as other theoretical results in the field (Stephas 1983, Recami 1982, 1984, 1985, 1986). In particular it is consistent with the well known fact that a free bradyon can trap no more than three free tachyons (Corben 1978b) or, in the wave picture, that no more than three space-like states can be superimposed on one time-like state giving another particle (Recami 1986 p 113, cf also Hoh 1976, Pagels 1976, Preparata 1976).

4. Time-like outer fields

Now, let us focus our attention on the problem of S^N-cavity outer fields governed by the propagation law (7). A detailed analysis of the problem indicates that the solutions of the Maxwell and the Helmholtz equations can be classified with respect to the S^N-cavity dimensions. Namely, for

(i) $N = 0$, the wave equation (6) describes the propagation of the outer vector fields associated with massless transverse photons moving at the *luminal* velocity;

(ii) $N = 1, 2$, the Maxwell equation (7) reduces to the spatially two (one)-*dimensional time-like Proca equation describing transmission of massive outer vector* fields associated with longitudinal photons travelling at a subluminal velocity;

(iii) $N = 3$, the spatial transmission of outer fields vanishes, the equation (7) reduces to the time-dependent formula $(\partial_0^2 + m_{n_\alpha}^2)\psi(x^0) = 0$, describing propagation of outer fields associated with the massive electromagnetic S^3-cavity, along the ct-axis.

Because in four-dimensional spacetime the direction of the time-like fields propagation coincides with the worldline along which the associated object moves, the ct-axis appears incidentally as the worldline of the S^3-cavity. It suggests that the time-like Klein–Gordon equation for a free-moving electromagnetic S^3-cavity at a velocity v is to be obtained by the application of the time Lorentz transformation (Elbaz 1987), yielding the result

$$(\partial_0^2 + m_{n_\alpha}^2)\exp[\mathrm{i}(m_{n_\alpha}x^0] = 0 \longmapsto (\partial_\mu\partial^\mu + m_{n_\alpha}^2)\exp[\mathrm{i}(k_\mu x^\mu] = 0 \qquad (17a, b)$$

$$k^\mu \equiv \left(\frac{m_{n_\alpha}}{\sqrt{1 - v^2c^{-2}}}, \frac{m_{n_\alpha}c^{-1}v}{\sqrt{1 - v^2c^{-2}}}\right). \qquad (18)$$

Simultaneously under uniform motion of the S^3-cavity, the inside trapped radiation is Doppler shifted (Jennison and Drinkwater 1977, Jennison 1978) and transforms into a system of time-like and space-like waves (Molski 1991) governed by the propagation laws

$$(\partial_\mu\partial^\mu + m_{n_\alpha}^2)\psi(x^\mu) = 0 \qquad (\partial_\mu\partial^\mu - m_{n_\alpha}^2)\psi(x^\mu)' = 0. \qquad (19)$$

The results obtained evidently indicate that the time-like Klein–Gordon equation is valid inside as well as outside the S^3-cavity, in full agreement with Elbaz (1987); however, they do not point for or against the existence of the space-like matter waves in the external S^N-cavity domain.

5. Time-trapped electromagnetic fields

From a methodological point of view, the derivation of the time-like Klein–Gordon equation for outer fields associated with a massive electromagnetic S^3-cavity, has been divided into two stages:

(i) on the basis of the Maxwell and Helmholtz equations, one obtains the time-dependent wave equation (17*a*);

(ii) application of the time Lorentz transformation leads to the final form of time-like Klein–Gordon equation (17*b*).

Because the *bradyon–tachyon* symmetry is strictly related to the *time–space* symmetry (Recami 1986 pp 34, 35), it would be interesting to obtain the space-like Klein–Gordon equation by taking advantage of the methodologically *mirror* operations of the time imprisonment of electromagnetic fields and, next, the application of the space Lorentz transformation.

Although, the possibility of time-trapped fields seems to be speculative, it deserves some attention as it plays a very important role in the quantum theory of space-like states. For example, Horodecki (1988b) considered in the non-relativistic regime the time quantization of space-like fields trapped in an impulse-like rectangular well, and Vyšin (1977) investigated the quantization of space-like states on a closed time line.

It is well known that the time- and space-like representations of the Poincaré group are SO_3 and $SO_{2,1}$ (Barut 1978), respectively; as a consequence, tachyons are not localizable in ordinary space (Peres 1970, Cawley 1970, Duffey 1975, 1980, Vyšin 1977) and appear to be more similar to fields than to particles of finite extension (Recami 1986 p 59). Tachyons always admit a particular class of subluminal reference frames (the so-called *critical frames*) from where they appear with speed $V = \infty$, i.e. as points in time simultaneously extended in space along a line (Recami 1986 p 56). In the light of the above facts the concept of tachyons as time cavities with trapped electromagnetic fields seems to be reasonable from the physical point of view, and worth studying. However, we shall not comment upon how such time inprisonment may be done practically.

In order to realize the above concept, let us consider a one-dimensional time cavity (T^1-cavity) of dimension T with trapped radiation. The Maxwell equation for this case reads

$$\partial_\mu \partial^\mu \phi_{n_0}(x^0) \exp[\mathrm{i}(k_\mu x^\mu)] = 0 \qquad \mu \neq 0 \tag{20}$$

where the functions $\phi_{n_0}(x^0)$ are solutions of the time-dependent Helmholtz equation

$$(\partial_0^2 + \mu_{n_0}^2)\phi_{n_0}(x^0) = 0 \qquad n_0 = 0, 1, 2 \ldots. \tag{21}$$

Taking into account time boundary conditions, similar to the spatial ones, the solutions of the Helmholtz equation (21) may be given as

$$\phi_{n_0}(x^0) = H_0 \cos(\pi n_0 x^0/T) \tag{22}$$

$$\phi_{n_0}(x^0) = E_0 \sin(\pi n_0 x^0/T) \tag{23}$$

$$\mu_{n_0}^2 = \pi^2 n_0^2/T^2 \tag{24}$$

whereas for the radiation trapped on the closed time line one obtains

$$\phi_{n_0}(x^0) = \exp[\mathrm{i}(2\pi n_0 T^{-1} x^0)] = \exp[\mathrm{i}(n_0 a_0^{-1} x^0)] \tag{25}$$

where a_0 is the radius of a circular time trajectory. If we assume that the rest mass

$$\mu_{n_0}^0 = \hbar \mu_{n_0} c^{-1} = \hbar \pi c^{-1} n_0 / a_0 \tag{26}$$

is associated with time-trapped radiation, then the Maxwell equation (20), for outer fields associated with T^1-cavity, reduces to

$$(\Delta + \mu_{n_0}^2) \exp[\mathrm{i}(\boldsymbol{k}\boldsymbol{r})] = 0 \tag{27a}$$

$$|\boldsymbol{k}| = \mu_{n_0}^0 c \hbar^{-1} \tag{27b}$$

and the application of the space Lorentz transformation to (27*a*) (Elbaz 1985, 1986, 1987) yields the space-like Klein–Gordon equation (Feinberg 1967)

$$(\Delta + \mu_{n_0}^2)\exp[\mathrm{i}(\boldsymbol{k}\boldsymbol{r})] = 0 \longmapsto (\partial_\mu \partial^\mu - \mu_{n_0}^2)\exp[\mathrm{i}(k'_\mu x^\mu] = 0 \tag{28}$$

$$k'^\mu \equiv \left(\frac{\mu_{n_0}}{\sqrt{v'^2c^{-2}-1}}, \frac{\mu_{n_0}c^{-1}\boldsymbol{v}'}{\sqrt{v'^2c^{-2}-1}}\right). \tag{29}$$

describing the propagation of outer fields, associated with a free-moving T^1-cavity, at a superluminal velocity v'. Such time-trapped electromagnetic fields are endowed with some tachyonic properties; for example, ponderability, delocalization in space and superluminal kinematics. However, the presented approach does not reproduce the three-dimensional extension of tachyons, which seems to be a genuine property of all particles independently of their time- or space-like characteristics.

6. Conclusions

The proposed electromagnetic approach provides the possibility of constructing a wave–particle model of an extended massive particle, and permits a better understanding of its internal structure. Namely, time-like objects appear as three-dimensional cavities with space-trapped electromagnetic fields. Such imprisoned radiation undergoes conversion into a system of time-like and space-like waves which *lock* to form the photon-like wave. In the particle picture, it may be interpreted as a photon conversion into a bradyon–tachyon system, whose constituents are self-trapped in a relativistically invariant way, yielding a photon-type compound particle. The three-dimensional space cavity has bradyon-type characteristic, i.e. it behaves as a massive subluminal extended object associated with time-like outer fields interpreted as matter waves of first kind (B-waves).

Exploiting the *mirror* methodology one may build up in a similar manner an electromagnetic model of space-like objects, which may be considered as cavities with time-trapped electromagnetic fields. Such trapped radiation is endowed with a rest mass depending on the cavity dimensions, and the outer space-like fields associated with a cavity can be interpreted as matter waves of the second kind (D-waves).

Finally, let us fix our attention on the problem of cavity deformation under uniform relativistic motion. Considering, for clarity's sake, S^1- and T^1-cavities travelling in the $M(1,1)$ space along the x-axis at the velocity v and v', respectively, from (18) and (29), one obtains

$$E = \frac{\pi n_1 c^2}{a_1\sqrt{1-v^2c^{-2}}} \qquad p = \frac{\pi n_1 v}{a_1\sqrt{1-v^2c^{-2}}} \tag{30}$$

$$E' = \frac{\pi n_0 c^2}{a_0\sqrt{v'^2c^{-2}-1}} \qquad p' = \frac{\pi n_0 v'}{a_0\sqrt{v'^2c^{-2}-1}}. \tag{30}$$

Now, it is apparent that dimensions a_0 and a_1 undergo relativistic deformation in the direction of the cavity motion, leading to the transformations

$$a_1 \to a_1\sqrt{1-v^2c^{-2}} \qquad a_0 \to a_0\sqrt{v'^2c^{-2}-1}. \tag{32}$$

Hence, the S^1-cavity grows shorter whereas the T^1-cavity is stretched out, in strict connection with the change of the associated mass

$$m_{n_1} \to m_{n_1}(1 - v^2c^{-2})^{-1/2} \qquad \mu_{n_0} \to \mu_{n_0}(v'^2c^{-2} - 1)^{-1/2}. \qquad (33)$$

In the light of these facts, the electromagnetic approach permits the relativistic mass problem to be considered in a purely geometrical framework, and explains why the mass of space-like objects diminishes under superluminal motion, in contradiction to *ordinary* particles, whose mass increases with velocity according to formula (33).

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